## JACAMaC Individual Algebra Questions - 10 Questions, 45 minutes, No Calculator. The point values for each question are in parentheses.

- 1. (3) If half of a number is 5 more than a third of the same number, what is the number?
- 2. (3) Points P and R are located at (0,3) and (8,17) respectively. Point M is the midpoint of segment  $\overline{PR}$ . Segment  $\overline{PR}$  is reflected over the y-axis. What is the sum of the coordinates of the image (reflected point) of point M?
- 3. (3) What is the slope of a line perpendicular to the line 3x 2y = -23? (The slope can be expressed in the form  $\frac{a}{b}$ ; express your answer as labl. Example: Express the answer of  $-\frac{17}{14}$  as 1714.)
- 4. (3) John is choosing between which rental car he should use to travel to New York. The pickup truck holds 25 gallons of gas but only gets 15 miles per gallon. The minivan holds 17 gallons of gas and gets 23 miles per gallon. The sedan holds 12 gallons of gas and gets 35 miles per gallon. The smart car holds only 8 gallons of gas but gets 50 miles per gallon. What is the farthest distance John can travel (in miles) on a full tank using one of the rental cars?
- 5. (4) 10 years ago, John's grandfather was twice the age of John's father, who in turn was twice the age of John. John is 63 years younger than his grandfather. How old is John today?
- 6. (4) Solve for x:  $x = \sqrt{506 + \sqrt{506 + \sqrt{506 + \dots}}}$

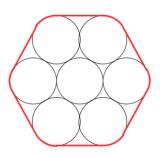
- 7. (5) Joe is deciding between 3 cell phone service providers: US Mobile, AT&G, and Horizon Wireless. US Mobile charges \$0.10 per minute of phone calls, \$0.15 per text, and \$0.75 per minute of internet wifi use. AT&G charges \$0.20 per minute of phone calls, \$0.05 per text, and \$0.50 per minute of internet wifi use. Horizon Wireless charges \$0.25 per minute of phone calls, a flat rate of \$20 per month for unlimited texting, and \$0.20 per minute of internet wifi use. Joe averages 30 minutes of phone calls, 250 texts, and 2 hours of internet wifi use per month. Assuming Joe goes with the service provider that will give him the least cost (based on his average cell phone use), how much will his average phone bill be? (The cost can be expressed in the form  $a + \frac{b}{100}$ ; leave your answer as ab. Example: Express the answer of \$12.63 as 1263.)
- (5) A store sells dorks, torques, and storks. Bob bought 3 torques and 5 dorks for \$25. Kathy bought 8 storks, 21 dorks, and 3 torques for \$153. Rich Joe bought 100 dorks and 2000 torques while returning (and being refunded for) 2 storks for a net total of money spent of \$10176. How much would Billy pay for 1 stork, 1 torque, and 1 dork?
- 9. (6) At 12:00, Tyler is the 324th person in line to ride the "Train of the Insane" rollercoaster. Each roller coaster train holds 32 people. A full train leaves every 3 minutes. If the first 32 people in line leave on the 12:01 train, what time will Tyler's train leave? (The time can be expressed in the form a:b; express your answer as ab. Example: Express the answer of 7:00 as 700.)
- 10. (6) Braxton enters a classroom at exactly 10:00 am and notices that the 12-hour analog clock on the wall is behaving strangely. The clock reads 12:00 and the second hand makes a complete rotation every 6 seconds. The minute hand and hour hand continue to behave as if every full rotation of the second hand indicates that a minute has passed. When Braxton leaves the class at 11:30 am, what time will the wall clock read? (The time can be expressed in the form a:b ; express your answer as ab. Example: Express the answer of 7:00 as 700.)

## JACAMaC Individual Geometry Questions- 10 Questions, 45 minutes, no calculator, point values of questions are in parenthesis.

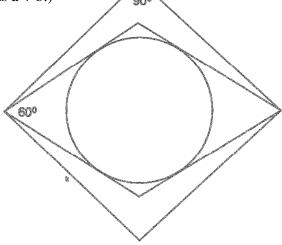
1. (3) An equilateral triangle and a regular hexagon both have the same side length. What is the ratio of the area of the triangle to the ratio of the hexagon? (For an answer  $\frac{a}{b}$  express

your answer as ab. Ex. Express the answer of  $\frac{17}{14}$  as 1714.)

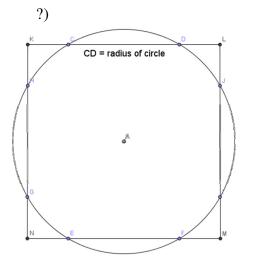
- 2. (3) A farmer wants to build a rectangular fence that has a length to width ratio of 2:3. He can use up to 75 feet of fencing. What is the maximum area which he can fence in? (Round your answer to the nearest whole number.)
- 3. (3) A circle with radius 10 is surrounded by and tangent to 6 congruent circles, also with radius 10 each. Each of the 6 circles is tangential to 2 of the other 5 circles as well as the center circle. A belt wrapped tightly around the outside of the 6 circles, as shown. What is the length of the belt? (The length can be expressed as  $a + b\pi$ ; what is a + b?)



4. (3) A circle is inscribed in a rhombus with one angle measuring 60 degrees. Another rhombus with side length 16 and an angle measuring 90 degrees is circumscribed around the first rhombus so that 2 vertices of the first rhombus touch 2 vertices of the second rhombus. Find the radius of the circle. (The radius can be expressed in the form  $a\sqrt{b}$ ; express your answer as a + b.)



5. (4) A circle and a square intersect so that each side of the square contains a chord of the circle equal in length to the radius of the circle. What is the ratio of the area of the circle to the area of the rectangle? (The ratio can be expressed in the form  $\frac{a\pi}{b}$ ; what is a + b



6. (4) An isosceles trapezoid is inscribed in a semicircle as shown below, such that the 3 shaded regions are congruent. The radius of the semicircle is 2. What is the area of the trapezoid? (The area can be expressed in the form  $a\sqrt{b}$ ; what is  $a \times b$ ?)



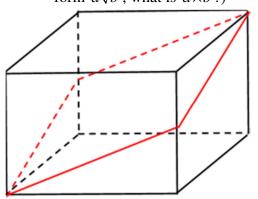
- 7. (5) A rectangular prism has a volume of 405 units<sup>3</sup>. The ratio of length : width : height of the prism is 1:3:5. What is the prism's surface area?
- 8. (5) A 5-unit line segment rotates 90° about one of its endpoints every second. The location of that endpoint moves one unit to the left every second. How much area is swept out by the line segment during the course of 4 seconds? (Area swept out is defined by all area covered by the line during its course, including the moment it started and the moment it ended. The area can be expressed as the product of *a* and  $\pi$  what is *a*?)

9. (6) A hexagonal pyramid and a triangular prism have the same volume. The hexagonal base of the pyramid is equilateral and equiangular, and the height of the pyramid is equal to the length one of the hexagonal base's sides. The triangular faces of the prism are equilateral, and the height of the prism is equal to the length one of the triangular base's sides. What is the ratio of the side length of the pyramid's hexagonal face to the side

length of the prism's triangular face? (The ratio can be expressed in simplest form as  $\frac{\sqrt[n]{b}}{c}$ 

; what is  $a \times b \times c$ ?)

10. (6) A cube is sliced by a plane which goes through 2 opposite corners and the midpoint of 2 edges as shown. If the cube has edge length 2, what is the area of the rhombus formed by the intersection of the plane and the cube? (The area can be expressed in the form  $a\sqrt{b}$ ; what is  $a \times b$ ?)



# JACAMaC Individual General Questions - 10 Questions, 45 minutes, no calculator, point values for each question are in parentheses.

- 1. (3) Bob is thinking of a number. This number is equal to the sum of the first 5 positive integers that leave a remainder of 7 when divided by 13. What is Bob's number?
- 2. (3) What are the last two digits of  $5^{2013}$ ?
- 3. (3) The last two digits of a perfect square repeat in a pattern such that if 2 numbers add to a multiple of 50, each of their squares ends in the same two digits. (A perfect square such as 1, 4 and 9 are considered to end in 01, 04, and 09, respectively.) What is the largest number less than 100 whose perfect square ends in 89?
- 4. (3) The addition problem below has a unique solution. Each of the letters, A, E, L, M and S represents a different nonzero digit. What would the sequence MESA represent?

#### S E A L +S E A L L L A M A

5. (4) The positive, even integers are written consecutively in the pattern below. What integer will be the 6th entry in Row B?

Row A				8						20	
Row B			6		10				18		22
Row C		4				12		16			
Row D	2						14				

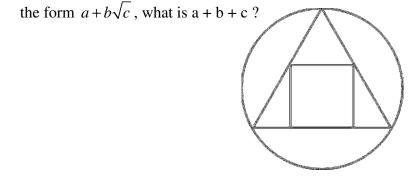
6. (4) Numbers on a standard 6-faced die are arranged such that numbers on opposite faces always add to 7. The product of the numbers appearing on the 4 lateral faces of a rolled die is calculated (ignoring the numbers on the top and bottom). What is the minimum possible value of this product?

- (5) For a certain natural number n, n<sup>2</sup> gives a remainder of 1 when divided by 5, and n<sup>3</sup> gives a remainder of 1 when divided by 5. What remainder does n give when divided by 5?
- 8. (5) Kathy has 20 quarters, 50 dimes, and 100 nickels. How many different ways can she make \$5 with only the coins she has?
- 9. (6) A video game developer wants to test their new game. In a certain level in their game, a player controls a character which exists in an environment with an infinite number of rooms. Each room has 5 tunnels leading out of it, 1 of which leads to the victory room, while the other 4 lead to other rooms. A player who enters the victory room instantly gains 50 points, and the level is passed. A player who enters a tunnel which does not lead to the victory room instantly loses 5 points, and once leaving a tunnel cannot find their way back. If a player starts the level with 100 points, how many points would they be expected (or estimated) to have after passing the level?
- 10. (6) Joe uses a spinner to select a number from 1 through 5, each with equal probability. Bob uses a different spinner to select a number 6 through 10, each with equal probability. What is the probability that the product of Joe's number and Bob's number is less than 20? (The probability can be expressed as the common fraction  $\frac{a}{b}$ ; express your answer as

ab. Example: Express the answer of  $\frac{14}{17}$  as 1417.)



- 1) Find the number of five-digit positive integers, *Tb*, that satisfy the following conditions:
  - (a) the number n is divisible by 5
  - (b) the first and last digits of *n* are equal, and
  - (c) the sum of the digits of m is divisible by 5
- 2) A circle is circumscribed around an equilateral triangle, and a square is then inscribed in the triangle. The area of the circle is  $36\pi$  units<sup>2</sup>. The area of the square can be written in



- 3) On a standard 3x3x3 Rubik's Cube with normal rotating axes, what is the maximum surface area that can be achieved in the optimal position? The surface area can be written in the form  $a + b\sqrt{c}$ ; what is a + b + c?
- 4) 1) How many integers between 1 and 1000, inclusive, can be represented as the difference of two integer squares?
- 5) Abby and Billy each have two fair six-sided dice. Each rolls their dice and note the product of their numbers. What is the probability that one person's product is exactly one third of the other?
- 6) In a video game, a player earns 100 points each time they complete a level. There are 5 different score multipliers a player can purchase: x2, x4, x6, x8, and x10, which cost 1000, 2000, 3000, 4000, and 5000 points, respectively. A player who has obtained a score multiplier has all points they earn from that time forward multiplied by a certain factor (for example, a player who has obtained the score x2 has every point they earn afterward doubled.) If a player has obtained multiple score multipliers, the factors they multiply points by are compounded (for example, if a player has obtained the score x2 and the score x6, all future points they earn will be multiplied by 2\*6 or 12). What is the least amount of levels a player would have to play in order to obtain all 5 score multipliers?

### JACAMAC Team Round Questions - 15 Questions, 30 minutes, maximum 4 people per team, answers go in crossword puzzle and each box is worth 1 point (29 points possible)

- 1. (Across) What is the greatest positive integer n such that  $5^n$  is a factor of 100!?
- 2. (Down) During the years between 1962 and 2013 (inclusive), there have been  $1.7 \times 10^9$  card games per year which have used a standard 52-card deck. Assuming that the probability plays out exactly, how many of these card games have had 4 aces (any order) in a row within a deck? (The amount can be expressed as a  $\times 10^6$ ; what is a ?)
- 3. (Across) A farmer named Xenu who lives on the planet Beeblebrox raises 2 types of creatures: Poggles and wingbats. Poggles are creatures with 4 legs and 2 eyes. Wingbats are creatures with 2 legs and 4 eyes. On Xenu's farm, there are a total of 66 legs and 72 eyes. How many creatures are on Xenu's farm?
- 4. (Across) A seven digit number, with a 7 placed to the right of it, is three times smaller than the same seven digit number with a 20 placed to the left of it. What is the seven digit number?

4. (Down) P(n) represents the probability that an "n" is rolled on a die. A 6-faced die, with faces 1 through 6, is weighted such that:

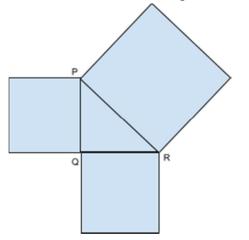
P(1) = P(3) P(2) = P(4) = 2(P(5)) P(4) = 3(P(3))P(5) = 2(P(6))

If this die is rolled once, what is the probability that a "5" is rolled? (For an answer  $\frac{a}{1}$ ,

express your answer as ab. Example: Express the answer of  $\frac{17}{14}$  as 1714.)

5. (Across) Each of the 5 digits 2, 3, 7, 8, 9 is placed in one of the boxes to form a mixed fraction as shown below. What is the largest value of all the mixed fractions that can be formed (assuming that when the values are placed, the numerator in the fractional part is not larger than the denominator)? (The number can be expressed as the common fraction  $\frac{a}{b}$ ; express your answer as ab. Example: Express the answer of  $\frac{17}{14}$  as 1714.)

- 6. (Down) If  $x + \frac{1}{x} = 5$ , then what is the value of  $x^3 + \frac{1}{x^3}$ ?
- 7. (Across) Points A(-1,-2), B(5,-2), C(5,8), and D(-1,8) are vertices of rectangle ABCD, and E is on segment CD at (2,8). What is the ratio of the area of triangle ADE to the area of quadrilateral ABCE? (The ratio can be expressed as the common fraction  $\frac{a}{b}$ ; express your answer as ab. Example: Express the answer of  $\frac{17}{14}$  as 1714.)
- 8. (Down) The function  $a \Diamond b$  is defined as  $a^b + b^a$  What is  $3 \Diamond 5$ ?
- 9. (Down) Given the equations 2x + 3y = 18, 2x + 3y + 4z = 17, and 3x + y + 3z = 42, what is the value of the sum x + y + z?
- 10. (Down) Angle PQR is a right angle. The 3 quadrilaterals shown are squares. The sum of the areas of the 3 squares is 144. What is the area of the largest square?



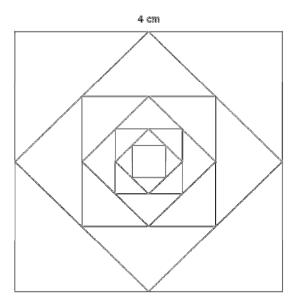
10. (Across) How many full 7-day weeks are there in 14 consecutive years? (Assume the 1st day of the 1st year is the 1st day of the week, and that there are no leap years.)

11. (Down) In a 4-digit number, the thousands digit is divisible by 3, the hundreds digit is greater than 4, the tens digit is greater than 2 but at most 6, and the ones digit is a prime number less than 10. How many 4-digit numbers satisfy these conditions?

12. (Across) A square is to have each of its corners randomly labeled A, E, M, and S. What is the probability that, when starting at the corner labeled M, and going counterclockwise or clockwise, that it spells MESA? (The probability can be expressed as the common

fraction  $\frac{a}{b}$ ; express your answer as ab. Example: Express the answer of  $\frac{17}{14}$  as 1714.)

13. (Down) The side length of a square is 4 cm long. Lines are drawn between the midpoints of each side to form a second square. The process of joining the midpoints of the sides of the innermost square is repeated. What is the perimeter of the 7th square? Add your answer to the area of the 1st square.



TEAM #\_\_\_\_\_

SCHOOL\_\_\_\_\_

